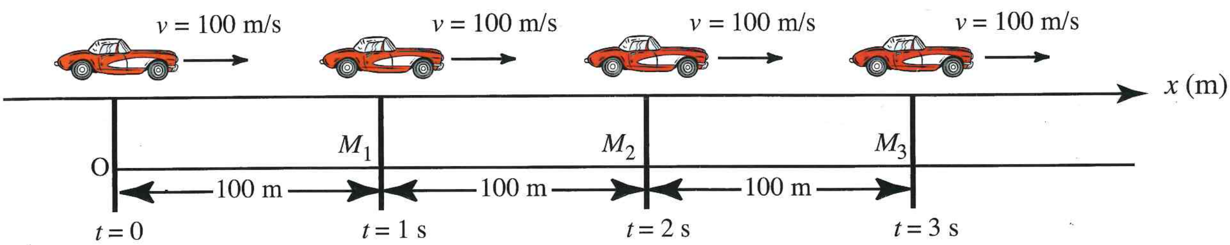
2.4 Motion with constant velocity

When a body moves with a constant velocity, it maintains a constant speed in a specific direction. Such kind of motion is called **uniform rectilinear motion**, abbreviated **URM**. In a URM, the moving body undergoes equal displacements in equal time intervals, hence, its position changes linearly with time. For instance, the position of a car cruising at 40 m/s in one direction, increases by 40 m every second. If the positive sense of motion is that of the plane and the position of the plane at $t = 0$ is chosen as origin (reference), then the position x of the plane at time t is simply $x = 40t$. At $t = 0$, $x = 0$; at $t = 1$ s, $x = 40$ m; at $t = 2$ s, $x = 80$ m; at $t = 3$ s, $x = 120$ m, etc.



The position of a body moving with a constant velocity is given by

$$x = vt + x_0$$

where v is the velocity and $x = x_0$ is the position at $t = 0$ s. A formal derivation of this rule will be given later on in this chapter.

Example 2.2

A body moves along a directed axis at a constant velocity of 2.5 m/s. At $t = 0$, the position of the body was 10.0 m.

Find the position of the body at $t = 3.0$ s if the body is moving a) in the positive sense, and b) in the negative sense.

Solution

For constant velocity, $x = vt + x_0$

a) $v = 2.5$ m/s and $x_0 = +10$ m, hence $x = 2.5t + 10$.
For $t = 3.0$ s, $x = (2.5 \text{ m/s})(3.0 \text{ s}) + 10 \text{ m} = 17.5$ m.

b) $v = -2.5$ m/s and $x_0 = +10$ m, hence $x = -2.5\,t + 10$.
For $t = 3.0$ s, $x = (-2.5 \text{ m/s})(3.0 \text{ s}) + 10 \text{ m} = +2.5$ m.

Activity

3 At an instant t_0, taken as a time reference, two cars A and B, moving in opposite directions with a constant velocity of 25 m/s, are spotted, one at 10 m to the right of an observer moving to right and the other at 100 m also to the right of the observer but moving to the left.

a) Choose the positive sense of motion to the right and the position of the observer as origin.
b) Express the position of A as a function of time.
c) Express the position of B as a function of time.
d) Determine the time at which A and B meet.
e) Determine the position where the two cars meet.

Exercises

8. Five seconds after she passes by Sally, a runner moving at a constant velocity passes by Mira standing 40 m away from Sally.
a) Write the equation of motion of the runner taking the position of Mira to be the origin of time and position.
b) Find the position of the runner at $t = 10$ s.

9. A body moving with constant velocity was at $x_1 = +12.5$ m when $t_1 = 3.0$ s, and at $x_2 = +25.0$ m when $t_2 = 5.5$ s. Find the equation of motion of the body and deduce its velocity.

4 **10.** A body is moving with a constant velocity along a directed straight line. At $t_1 = 2.0$ s the position of the body is $x_1 = 8.4$ m and at time $t_2 = 7.0$ s the position of the body is $x_2 = -1.6$ m.
Find the equation of motion and the velocity of that body.

Rewriting the equation of motion

The equation $x = vt + x_0$ may be rewritten as $x - x_0 = vt$ or $\Delta x = vt$. It follows that in a URM the displacement in a time interval t is the product of its velocity by that time.

Example 2.3

Find the displacement of a particle moving with a velocity of 20.0 $m.s^{-1}$ for 3.0 seconds.

Solution

The displacement is given in this case by $\Delta x = vt = 20t$.

For $t = 3.0$ s, $\Delta x = (20\ m.s^{-1})(3.0\ s) = 60$ m, hence the displacement is 60 m in the same sense as the velocity.

In a URM the displacement in a time interval Δt can be determined using $\Delta x = v\Delta t$; this is independent of the choice of the origin of time and position.

For instance, if the position of the particle in the above example is given by $x = 20t - 5$ ($v = 20$ m/s, and $x_0 = -5$ m), we may find the displacement in the time interval [2 s, 5 s] in two different ways:

i. At $t = 2$ s, $x = (20\ m/s)(2\ s) - (5\ m) = 35$ m and at $t = 5$ s, $x = (20\ m/s)(5\ s) - (5\ m) = 95$ m hence, the displacement is $\Delta x = x_f - x_0 = 95\ m - 35\ m = 60$ m.

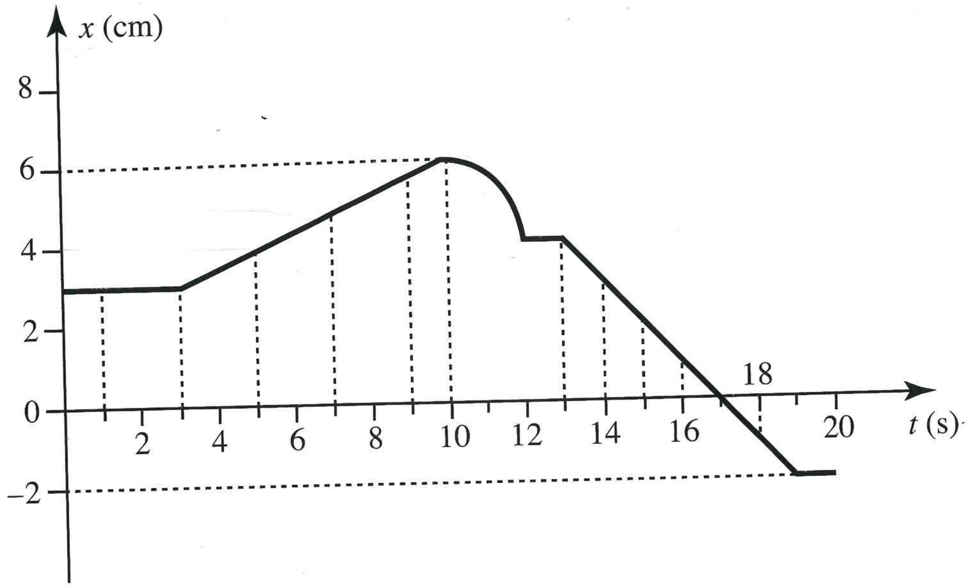
ii. The displacement is given by $\Delta x = v\Delta t = 20\Delta t$. In the interval [2 s, 5 s], $\Delta t = 3$ s, hence, $\Delta x = (20\ m/s)(3\ s) = 60$ m.

2.5 Position-time graphs

One dimensional motion can be described in words, using equations, *motion diagrams* or graphs. A position-time graph is a plot of 1-D position x versus time t. This means that t is the independent variable and x is the dependent variable. We therefore plot time along the (so-called) horizontal axis and position along the (so-called) vertical axis. We may say that, for a particle, position x is a function of time t.

Activity

The x-t graph shown below represents the motion of a particle moving along a horizontal axis in 20 s.



a) Complete the following table

t (s)	0.0	1.0	3.0	5.0	7.0	9.0	10	13.0	14.0	15.0	16.0	17.0	18.0
x (cm)	3.0	3	3.0	4	5	5.5	6.0	4.0	3	2	1	0	−1

b) Determine the time interval during which the particle position does not change.

c) What can we say about the speed of the particle in that time interval?

d) What is the farthest point from the origin? What is the corresponding time?

e) At what instant(s) is the puck at the origin?

f) In which interval(s) is the puck

1- moving away from the origin?

2- moving towards the origin?

g) Use a number line to locate the different positions of the puck.

Students should not get confused between x-t *graph* and *trajectory*. In the above example, the body is initially at $x = 3$ m. After 3 seconds, it moves in the +ve sense to position $x = 6$ m taking 7 s to reach there. At that instant it stops spontaneously before it moves back to $x = 4$ m where it stays there for 1 second, it then continues its motion in the negative sense, continues to $x = -2$ m where it stops after crossing the origin at $t = 17$ s. The trajectory is thus the line segment $[-2 \text{ m}, 6 \text{ m}]$, which is the projection of the graph on the x-axis.