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الوطنية الأرثوذكسية
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The National
Orthodox School
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Worksheet 6 |

1st Semester | 2023-2024

Subject: Math   **Chapter:** 3

Objectives:
- To work with simple powers and roots using mental methods
- To work with fractional and negative powers, and reciprocals
- To use the laws of indices to simplify numerical expressions

The laws of indices:

$$a^m \times a^n = a^{m+n} \qquad (1)$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad (2)$$

$$(a^m)^n = (a^n)^m = a^{mn} \qquad (3)$$

$$a^{-m} = \frac{1}{a^m} \qquad (4)$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m \qquad (5)$$

$$a^0 = 1 \qquad (6)$$

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad (7)$$

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} \qquad (8)$$

$$\sqrt{a} \times \sqrt{a} = a \qquad (9)$$

where **_a, m_** and **_n_** are real numbers.

Notes on powers with negative integers:

1- A negative base raised to an **odd** power will **always** be negative!

Example:

$$(-1)^3 = -1 \times -1 \times -1 = -1$$

$$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$$

$$(-3)^3 = -27$$

2- A negative base raised to an **even** power will **always** be **positive**!

$$(-1)^2 = -1 \times -1 = 1$$

$$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$$

3- Note **brackets** when expanding the exponents.

Examples:

$$-2^2 = -(2 \times 2) = -4 \quad \textit{the square is only for the two.}$$

$$(-2)^2 = -2 \times -2 = 4 \quad \textit{the square is for negative two.}$$

4- The power can be **distributed** over multiplication and division but **NOT** over addition and subtraction.

Examples:

$$(3y)^3 = 3^3 y^3 = 27 y^3$$

$$(\frac{2}{x})^2 = \frac{2^2}{x^2}$$

$$(3+y)^2 = (3+y)(3+y) \neq 3^2 + y^2$$

$$(y-5)^2 = (y-5)(y-5) \neq y^2 - 5^2$$

Q1: Find the exact values of:

$\sqrt[3]{8} =$	$\sqrt[3]{-27} =$
$\sqrt[3]{\dfrac{1}{8}} =$	$\sqrt[3]{\dfrac{-8}{64}} =$
$\sqrt[3]{5} \times \sqrt[3]{25} =$	$\sqrt[3]{3\dfrac{3}{8}} =$

Q2: Find x:

$x^2 = \dfrac{4}{81}$	$2x^2 = 200$
$x^3 = 8$	$x^3 = -27$
$x^3 = 0$	$2x^3 - 50 = 200$

Q3: Find the exact values of:

$9^{\frac{1}{2}} \times 8^{\frac{1}{3}}$

$16^{\frac{3}{2}}$

$8^{-\frac{2}{3}}$

$\left(\dfrac{3}{4}\right)^{-2}$

$\sqrt{(2 \times 2 \times 3 \times 3 \times 5 \times 5)}$

Q4: Find the exact values of:

$$a^{\frac{1}{2}} \times a^{\frac{1}{2}} =$$

$$a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} =$$

$$a^{\frac{1}{4}} \times a^{\frac{1}{4}} \times a^{\frac{1}{4}} \times a^{\frac{1}{4}} =$$

Q5: Find the exact values of:

a) $64^{\frac{1}{3}}$	b) $125^{\frac{1}{3}}$	c) $25^{\frac{3}{2}}$
d) $8^{\frac{5}{3}}$	e) $16^{\frac{3}{4}}$	f) 4^{-3}
g) $4^{-\frac{1}{2}}$	h) $27^{-\frac{2}{3}}$	i) $9^{-\frac{1}{2}} \times 2^3$
j) $(3e)^0$	k) $8^{-\frac{2}{3}}$	l) $\left(\dfrac{25}{16}\right)^{-\frac{3}{2}}$
m) $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}}$	n) $(\sqrt{3})^2$	o) $\sqrt{2^4 \times 9}$

Q6: Simplify $\left(\dfrac{64x^6}{25y^2}\right)^{-\frac{1}{2}}$

Q7:
a) Simplify, leaving your answer in index form
   i) $4^7 \times 4$ ii) $6^3 \div 6^5$

b) $7^y = 1$
   Find the value of y.

Q8: Evaluate the following.
Give your answers as fractions.
a) 3^{-4}

b) $\sqrt[3]{\left(\dfrac{8}{27}\right)^2}$

c) $\left(\dfrac{\sqrt{6}}{4}\right)^4$

Q9:
a) Find, as a fraction, the value of $\dfrac{4+2^3}{(4+2)^2}$

b) Write your answer to part a) as a decimal.

Q10: $3^n = \dfrac{1}{9}$

Find the value of n.

Q11: $3 \times \sqrt{27} = 3^n$

Find the value of n.

Q12: **a)** Simplify, leaving your answer in index form

i) $5^4 \times 5^8$ **ii)** $9^7 \div 9^4$

b) Solve $\dfrac{3^8 \times 3^5}{3^n} = 3^4$